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A Class of Estimators for Mean of Symmetrical Population when the Variance is not known

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SUMMARY

A class of estimators of population mean (μ) when the variance (σ^2) is unknown, is proposed in case of symmetrical populations. Bias and mean square error are found for the class. Various estimators are shown to belong to the class and a sub-class of optimum estimators in the sense of having minimum mean square error is found.
Keywords : Class of estimators, Coefficient of variation, Mean square error and optimum estimators, Unknown variance.

Introduction

Utilising known square of coefficient of variation $C^2\left(=\frac{\sigma^2}{\mu^2}\right)$, Searles [2] proposed an improved estimator of population mean μ; but when C^2 is unknown, the problem of estimation consists of estimators using the estimates of C^2 given by

$$\hat{C}^2 = \frac{s^2}{\bar{y}^2} \qquad \text{or} \qquad \hat{C}^2 = \frac{s^2}{\bar{y}^2}\left(1-\frac{s^2}{n\bar{y}^2}\right)^{-1}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $s^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(y_i-\bar{y})^2$ for the values $y_1, y_2, \ldots, y_n$ of a random sample of size n.

In this paper, with $u = \frac{s^2}{n\,\bar{y}^2}$, we propose the following class of estimators for population mean μ

$$t = f\left(\bar{y}, s^2/n\,\bar{y}^2\right) = f(\bar{y}, u)$$

where $f(\bar{y}, u)$ satisfying the validity conditions of Taylor's series expansion, is the function of $(\bar{y}, u)$ such that $f(\mu, 0) = \mu$, first order partial derivative

$$W_{1h} = 1 - W_{2h}$$

To obtain the optimum values of n′, v_h and k_h we adopt a stepwise minimization technique. First using Lagrange's multiplier we minimize the variance of $\bar{y}^*_{ds}$ (see (2.5)) subject to the fixed expected cost C* given in (5.2). This results in the optimum value of k_h given by

$$k_{0h} = \frac{1}{2}\{(C_{22h}(1 - S^2_{2yh}))^2 + 4C_{22h}S^2_{2yh}C_h\Delta_h\}^{\frac{1}{2}}/S^2_{2yh}C_h \tag{5.3}$$

where $C_h = C_{2h}W_h + C_{21h}W_{1h}$

$$\Delta_h = W_hS^2_{yh} - W_{2h}S^2_{2yh}$$

By plugging k_{0h} in (5.2) and (2.5) and following Cochran [1] the optimum value of v_h is

$$v_{0h} = \{C_1(\Delta_h + W_{2h}k_{0h}S^2_{2yh})\}^{\frac{1}{2}} \div \{(S^2_y - \Sigma W_hS^2_{yh})(C_h + C_{22h}W_{2h}/k_{0h})\}^{\frac{1}{2}} \tag{5.4}$$

The optimum n′ is hence obtained for either fixed cost or fixed variance using (5.2) or (2.5). For e^*_{DC}, the optimum values of k_h and v_h are obtained by replacing S^2_{yh} in (5.3) and (5.4) with $S^2_{yh} + \lambda^2S^2_{xh} - 2\lambda S_{xyh}$. While for e^*_{RC} and e^*_{DS}, S^2_{yh} is replaced with $S^2_{yh} + R^2S^2_{xh} - 2RS_{xyh}$ and $S^2_{yh} + \lambda^2_hS^2_{xh} - 2\lambda_hS_{xyh}$ respectively.

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$f'_1 = \left.\frac{\delta f(\bar{y}, u)}{\delta \bar{y}}\right]_{(\mu, 0)} = 1$, second order partial derivative $\frac{\delta^2 f(\bar{y}, u)}{\delta \bar{y}^2} = 0$ and second order partial derivative $f''_{12} = \left.\frac{\delta^2 f(\bar{y}, u)}{\delta \bar{y}\, \delta u}\right]_{(\mu, 0)} = \frac{f'_2 \delta}{\mu}$ (with f'_2 being the first order partial derivative of $f(\bar{y}, u)$ with respect to u at the point $(\mu, 0)$ and δ taking one of the two values 'zero and unity' depending upon the particular form of an estimator. For example, for the estimator $\bar{y} + ku$, δ takes value zero whereas for the estimator $\bar{y}(1-u)$, $\delta = 1$.

Some special cases of the generalized estimator t when σ^2 unknown and k, g, α being the characterising scalars, are

(1) $$t_1 = \bar{y} + k\frac{s^2}{n\bar{y}^2} = \bar{y} + ku$$

(2) $$t_2 = \bar{y} + k\frac{s^2}{n\bar{y}^2}\left(1 + g\frac{s^2}{n\bar{y}^2}\right) = \bar{y} + ku\,(1 + gu)$$

(3) $$t_3 = \bar{y}\left[1 + \frac{ks^2}{n\bar{y}^2}\left(1 + g\frac{s^2}{n\bar{y}^2}\right)^{-\alpha}\right] = \bar{y}\left(1 + ku(1+gu)^{-\alpha}\right)$$ by Singh [3]

(4) $$t_4 = \bar{y}\left[1 - \frac{s^2}{n\bar{y}^2}\left(1 + \frac{s^2}{n\bar{y}^2}\right)^{-1}\right] = \bar{y}\,[1 - u(1+u)^{-1}]$$ by Srivastava [4, 5]

(5) $$t_5 = \bar{y}\left(1 - \frac{s^2}{n\bar{y}^2}\right) = \bar{y}(1-u)$$ by Srivastava [4,5]

(6) $$t_6 = \bar{y}\left[1 + \frac{ks^2}{n\bar{y}^2}\left(1 - \frac{ks^2}{n\bar{y}^2}\right)^{-1}\right] = \bar{y}\left[1 + ku\,(1-ku)^{-1}\right]$$ by Thompson [9]

(7) $$t_7 = \bar{y}\left(1+\frac{s^2}{n\bar{y}^2}\right) = \bar{y}(1+u)$$ by Upadhyaya and Srivastava [10]

(8) $$t_8 = \bar{y}\left[1+\frac{s^2}{n\bar{y}^2}\left(1+\frac{s^2}{n\bar{y}^2}\right)^{-1}\right] = \bar{y}\left[1+u(1+u)^{-1}\right]$$ by Sahai and Ray [1]

(9) $$t_9 = \bar{y}\left[1+\frac{s^2}{n\bar{y}^2}\left(1+\frac{s^2}{n\bar{y}^2}\right)^{-2}\right] = \bar{y}\left[1+u(1+u)^{-2}\right]$$ by Srivastava and Banarsi [6]

(10) $$t_{10} = \bar{y}\left[1+\frac{ks^2}{n\bar{y}^2}\left(1+\frac{gs^2}{n\bar{y}^2}\right)^{-1}\right] = \bar{y}\left[1+ku(1+gu)^{-1}\right]$$ by Srivastava and Bhatnagar [7]

(11) $$t_{11} = \bar{y}\left[1+\frac{s^2}{n\bar{y}^2}\left(1-\frac{s^2}{n\bar{y}^2}\right)^{-1}\right] = \bar{y}\left[1+u(1-u)^{-1}\right]$$ by Srivastava and Dwivedi [8]

where various forms of the function $f(\bar{y}, u)$ are given by the expression on right hand sides of (1) to (11) in terms of $\bar{y}$ and u.

It may be mentioned here that all the estimators listed from (1) to (11) belong to the class t and satisfy the condition $f(\mu, 0) = \mu$ with $f'_1 = 1$ and $f''_{12} = f'_2\,\delta/\mu$, $\delta = 1$ or 0.

2. *BIAS AND MEAN SQUARE ERROR OF t*

To find the bias and mean square error (MSE) of t upto terms of order $0(n^{-2})$, let

$$\bar{y} = \mu + z \text{ amd } s^2 = \sigma^2 + v \tag{2.1}$$

where z and v are of order $0(n^{-1/2})$ with $E(z) = E(v) = 0$, and $E(z^2) = \frac{\sigma^2}{n} = \frac{\mu^2 C^2}{n}$.

With $\bar{y}^* = \mu + \theta(\bar{y} - \mu)$ and $u^* = \theta u$, $0 < \theta < 1$, expanding $t = f(\bar{y}, u)$ in third order Taylor's series about the point $(\mu, 0)$, we have

$$t = f(\mu, 0) + (\bar{y} - \mu) \left.\frac{\delta f(\bar{y}, u)}{\delta \bar{y}}\right]_{(\mu, 0)} + u \left.\frac{\delta f(\bar{y}, u)}{\delta u}\right]_{(\mu, 0)}$$

$$+ \frac{1}{2!}\left\{(\bar{y} - \mu)^2 \left.\frac{\delta^2 f(\bar{y}, u)}{\delta \bar{y}^2}\right]_{(\mu, 0)} + 2(\bar{y} - \mu) \,.\, u \left.\frac{\delta^2 f(\bar{y}, u)}{\delta \bar{y}\, \delta u}\right]_{(\mu, 0)}\right.$$

$$\left. + u^2 \left.\frac{\delta^2 f(\bar{y}, u)}{\delta u^2}\right]_{(\mu, 0)} \right\} + \frac{1}{3!}\left\{(\bar{y} - \mu)\frac{\delta}{\delta \bar{y}} + u \frac{\delta}{\delta u}\right\}^3 f(\bar{y}^*, u^*) \tag{2.2}$$

Now, we have $\left.\frac{\delta f(\bar{y}, u)}{\delta \bar{y}}\right]_{(\mu, 0)} = 1, \frac{\delta^2 f(\bar{y}, u)}{\delta \bar{y}^2} = 0,$

$\frac{\delta^3 f(\bar{y}, u)}{\delta \bar{y}^3} = 0, \frac{\delta^3 f(\bar{y}, u)}{\delta \bar{y}^2\, \delta u} = 0$; and further for $\bar{y} - \mu = z$ and $\left|\frac{z}{\mu}\right| < 1$,

$$u = \frac{s^2}{n\bar{y}^2} = \frac{\sigma^2(1 + v/\sigma^2)}{n\mu^2\left(1 + \frac{z}{\mu}\right)^2} = \frac{C^2}{n}\left(1 + \frac{v}{\sigma^2}\right)\left(1 + \frac{z}{\mu}\right)^{-2}$$

$$= \frac{C^2}{n}\left(1 + \frac{v}{\sigma^2}\right)\left(1 - \frac{2z}{\mu} + \ldots\right)$$

so that

$$t = \mu + z + \frac{C^2}{n}\left(1 + \frac{v}{\sigma^2}\right)\left(1 - \frac{2z}{\mu} + \ldots\right)f'_2$$

$$+ \frac{1}{2!}\left\{0 + 2z\frac{C^2}{n}\left(1 + \frac{v}{\sigma^2}\right)\left(1 + \frac{z}{\mu}\right)^{-2} f''_{12} + \frac{C^4}{n^2}\left(1 + \frac{v}{\sigma^2}\right)^2\left(1 + \frac{z}{\mu}\right)^{-4} f''_2\right\}$$

$$+ \frac{1}{3!}\left\{0 + 3(\bar{y} - \mu)^2 u \frac{\delta^2 f(\bar{y}^*, u^*)}{\delta \bar{y}^{*2}\, \delta u^*} + 3(\bar{y} - \mu) u^2 \frac{\delta^3 f(\bar{y}^*, u^*)}{\delta \bar{y}^*\, \delta u^{*2}}\right.$$

$$\left. + u^3 \frac{\delta^3 f(\bar{y}^*, u^*)}{\delta u^{*3}}\right\}$$

$$= \mu + z + \frac{C^2}{n}\left(1+\frac{v}{\sigma^2}\right)\left(1-\frac{2z}{\mu}+3\frac{z^2}{\mu^2}-4\frac{z^3}{\mu^3}+5\frac{z^5}{\mu^5}-\ldots\right)f'_2$$

$$+\frac{1}{2\,!}\left\{\frac{2z\,C^2}{n}\left(1+\frac{v}{\sigma^2}\right)\left(1-\frac{2z}{\mu}+\frac{3z^2}{\mu^2}-\frac{4z^3}{\mu^3}+\frac{5z^4}{\mu^4}-\ldots\right)f_{12}''\right.$$

$$\left.+\frac{C^4}{n^2}\left(1+\frac{v}{\sigma^2}\right)^2\left(1+\frac{z}{\mu}\right)^{-4} f_2''\right\}+\frac{1}{3\,!}\left\{0+3\,(\bar{y}-\mu)^2\,\frac{u\,\delta^2 f(\bar{y}^*,u^*)}{\delta\bar{y}^{*2}\,\delta u^*}\right.$$

$$\left.+3\,(\bar{y}-\mu)\,u^2\,\delta^3\,\frac{f(\bar{y}^*,u^*)}{\delta\bar{y}^*\,\delta u^{*2}}+u^3\,\delta^3\,\frac{f(\bar{y}^*,u^*)}{\delta u^{*3}}\right\}$$

$$=\mu+z+\frac{1}{n}\left\{\left(C^2-2C^2\frac{z}{\mu}+\frac{v}{\mu^2}+3C^2\frac{z^2}{\mu^2}-\frac{2z\,v}{\mu^3}\right)-4C^2\frac{z^3}{\mu^3}\right.$$

$$\left.+3\frac{z^2v}{\mu^4}+5C^2\frac{z^4}{\mu^4}-4\frac{z^4v}{\mu^6}+\ldots\right\}f'_2+\frac{1}{n}\left\{\left(C^2z-2C^2\frac{z^2}{\mu}+\frac{z\,v}{\mu^2}\right)\right.$$

$$+3C^2\frac{z^3}{\mu^2}-2\frac{z^2\,v}{\mu^3}-4C^2\frac{z^4}{\mu^3}+3\frac{z^3\,v}{\mu^4}+5C^2\frac{z^5}{\mu^4}-4\frac{z^4\,v}{\mu^5}$$

$$\left.+5\frac{z^5\,v}{\mu^6}+\ldots\right\}f''_{12}+\frac{C^4}{2n^2}\left(1+\frac{v}{\sigma^2}\right)^2\left(1+\frac{z}{\mu}\right)^{-4}f''_2$$

$$+\frac{1}{3\,!}\left\{3\,(\bar{y}-\mu)^2\,u\,\delta^3\,\frac{f(\bar{y}^*,u^*)}{\delta\bar{y}^{*2}\,\delta u^*}+3\,(\bar{y}-\mu)\,u^2\,\frac{\delta^3 f(\bar{y}^*,u^*)}{\delta\bar{y}^*\,\delta u^{*2}}\right.$$

$$\left.+u^3\,\frac{\delta^3 f(\bar{y}^*,u^*)}{\delta u^{*3}}\right\} \qquad (2.3)$$

Taking expectation in (2.3), to the terms of order $0\,(n^{-2})$ for symmetrical populations, we have

$$E\,(t) = \mu+E\,\frac{C^2}{n}\left(1+\frac{3z^2}{\mu^2}\right)f'_2-\frac{2z^2C^2}{n\mu}f''_{12}+\frac{C^4}{2n^2}f''_2$$

$$= \mu+\frac{C^2}{n}\left(1+\frac{3C^2}{n}\right)f'_2-\frac{2\mu C^4}{n}f''_{12}+\frac{C^4}{2n^2}f''_2$$

or $\quad$ $\text{Bias}\,(t) = E\,(t)-\mu$

$$= \frac{C^2}{n}\left[\left(1+\frac{C^2}{n}\right)f'_2+\frac{2C^2}{n}\left(f'_2-\mu\,f''_{12}\right)+\frac{C^2}{2n}f''_2\right] \qquad (2.4)$$

Again, from (2.3), we have

$$\text{MSE}(t) = E(t-\mu)^2$$

$$= E\Bigg[z + \frac{1}{n}\left\{ \left(C^2 - 2C^2 \frac{z}{\mu} + \frac{v}{\mu^2} + 3C^2 \frac{z^2}{\mu^2} - \frac{2zv}{\mu^3} \right) - 4C^2 \frac{z^3}{\mu^3} \right.$$

$$\left. + 3\frac{z^2 v}{\mu^4} + 5C^2 \frac{z^4}{\mu^4} - 4\frac{z^3 v}{\mu^5} + 5\frac{z^4 v}{\mu^6} + \dots \right\} f'_2$$

$$+ \frac{1}{n}\left\{ \left(C^2 z - 2C^2 \frac{z^2}{\mu} + \frac{zv}{\mu^2} \right) + 3C^2 \frac{z^3}{\mu^2} - \frac{2z^2 v}{\mu^3} - 4C^2 \frac{z^4}{\mu^3} \right.$$

$$\left. + 3\frac{z^3 v}{\mu^4} + 5C^2 \frac{z^5}{\mu^4} - \frac{4z^4 v}{\mu^5} + \frac{5z^5 v}{\mu^6} + \dots \right\} f''_{12}$$

$$+ \frac{C^4}{2n^2}\left(1 + \frac{v}{\sigma^2}\right)^2 \left(1 + \frac{z}{\mu}\right)^{-4} f'_2$$

$$+ \frac{1}{3!}\left\{ 3(\bar{y}-\mu)^2 u \frac{\delta^3 f(\bar{y}^*, u^*)}{\delta \bar{y}^{*2} \delta u^*} + 3(\bar{y}-\mu) u^2 \frac{\delta^3 f(\bar{y}^*, u^*)}{\delta \bar{y}^* \delta u^{*2}} \right.$$

$$\left. + u^3 \frac{\delta^3 f(\bar{y}^*, u^*)}{\delta u^{*3}} \right\} \Bigg]^2$$

whence, upto terms of order $0(n^{-2})$, the mean square error of t is

$$\text{MSE}(t) = E\left[z^2 + \frac{C^4}{n^2}(f'_2)^2 + \frac{2z}{n}\left(C^2 - 2C^2 \frac{z}{\mu} + \frac{v}{\mu^2} \right) f'_2 + 2\frac{z^2}{n} C^2 f''_{12} \right].$$

from which, for symmetrical populations, upto terms of order $0(n^{-2})$ the mean square of t is

$$\text{MSE}(t) = \frac{\mu^2 C^2}{n} + \frac{C^4}{n^2}(f'_2)^2 - 4\mu \frac{C^4}{n^2} f'_2 + \frac{2\mu^2 C^4}{n^2} f''_{12}$$

$$= \frac{\mu^2 C^2}{n}\left[1 + \frac{C^2}{n}\left\{ \frac{(f'_2)^2}{\mu^2} - \frac{4f'_2}{\mu} + \frac{2f'_2 \delta}{\mu} \right\} \right] \tag{2.5}$$

which is minimised for

$$f'_2 = \mu(2-\delta) \tag{2.6}$$

where δ takes one of the two values 'O and 1'; and the minimum mean square error is given by

$$\text{MSE}(t)_{\text{min.}} = \frac{\mu^2 C^2}{n}\left[1 + \frac{C^2}{n}\left\{(2-\delta)^2 - 4(2-\delta) + 2(2-\delta)\delta\right\}\right]$$

$$= \frac{\mu^2 C^2}{n}\left[1 - \frac{C^2}{n}(2-\delta)^2\right] \tag{2.7}$$

3. *Concluding Remarks*

(a) From (2.6) and (2.7), the class of estimators t attains its minimum value for $f'_2 = \mu(2-\delta), \delta = \mu f''_{12}/f'_2$ and the minimum mean square error is

$$\text{MSE}(t)_{\text{min.}} = \frac{\mu^2 C^2}{n}\left[1 - \frac{C^2}{n}(2-\delta)^2\right] \tag{3.1}$$

Thus, any estimator from the class t cannot have mean square error less than the expression given by (3.1).

(b) Bias, mean square error and the related results to the estimators listed in section 1 may easily be found as special cases of this study. For example, with k,g and α being the characterizing scalars for the estimator

$$t_3 = \bar{y}\left[1 + \frac{k s^2}{n \bar{y}^2}\left(1 + \frac{g s^2}{n \bar{y}^2}\right)^{-\alpha}\right]$$

$$= \bar{y}[1 + k u (1 + g u)^{-\alpha}]$$

by Singh [3], we have $f''_{12} = k, f'_2 = k\mu, \delta = \dfrac{\mu f''_{12}}{f'_2} = 1$

so that $f'_2 = \mu(2-\delta) = k\mu$ satisfying (2.6) gives the value of k=1 for which $\text{MSE}(t_3)$ is minimised and the minimum mean square error

$$\text{MSE}(t_3)_{\text{min.}} = \frac{\mu^2 C^2}{n}\left(1 - \frac{C^2}{n}\right) \tag{3.2}$$

is obtained from (3.1) by putting $\delta = 1$. Further, for the estimator t_3, we have $f'_2 = k\mu, f''_{12} = k$ and $f''_2 = -2\alpha k g \mu$ so that the bias of t_3 from (2.4) is

$$\text{Bias}\,(t_3) = \frac{k\,\mu\, C^2}{n}\left[1 + \frac{C^2}{n}(1 - \alpha\, g)\right] \tag{3.3}$$

which, for k= g= 1, reduces to

$$\text{bias}\,(t_3) = \frac{\mu\, C^2}{n}\left[1 - (\alpha - 1)\frac{C^2}{n}\right] \tag{3.4}$$

It may be mentioned here that the expressions (3.2) and (3.4) are the same expressions as obtained by Singh [3]. Similarly, the results of all the estimators listed in section 1 may easily be shown to be special cases of those of the generalized estimator t.

(c) For the estimators having $f''_{12} = 0$, that is $\delta = 0$ we have from (2.7)

$$\text{MSE}\,(t)_{\text{min.}} = \frac{\mu^2\, C^2}{n}\left(1 - \frac{4C^2}{n}\right) \tag{3.5}$$

For example, the estimator $t_1 = \bar{y} + k\dfrac{s^2}{n\,\bar{y}^2} = \bar{y} + k\,u$, k being the characterizing scalar, has $f'_2 = k$, $f''_{12} = 0$ and $\delta = 0$ so that it attains, for the optimum value $f'_2 = \mu\,(2 - \delta) = k$ satisfying (2.6) and giving $k = 2\mu$ the minimum mean square error given by (3.5).

(d) The estimator like t_3 has the practical advantage over the estimator like t_1, since the optimum value k =1 minimizing mean square error for t_3 is independent of parameter whereas the optimum value $k = 2\mu$ in case of t_1 depends upon the parameter μ. In fact, for the sub-set of estimators of the form $t_s = \bar{y}\, h\,(u)$ of the class t where h(u) is the function of u such that $h\,(0) = 1$, there is no practical difficulty in using the optimum value $f'_2 = \mu\, h'\,(0) = \mu$ (the value of δ for t_s is unity and $h'\,(0)$ is the first derivative of h(u) with respect to u at u =O) giving $h'\,(0) = 1$, a quantity independent of the parameter.

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